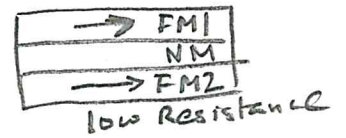




Giant Magnetoresistance



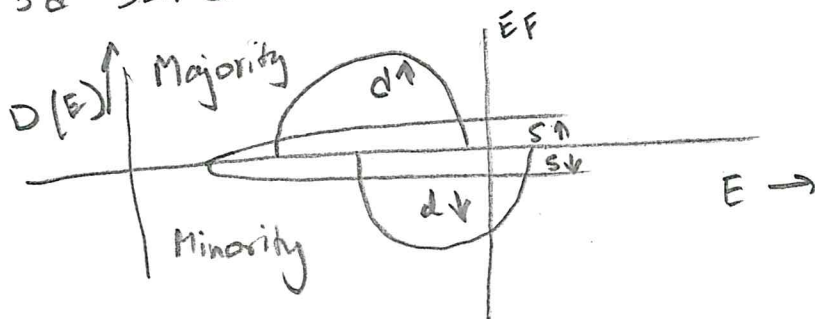
- Observed in magnetic multilayers such as $(\text{Fe/Cr})_n, (\text{Co/Cu})_n$
- Resistance difference between parallel and anti-parallel configs.
- In practice, only a certain combinations show high value.
- Best results obtained in $(\text{Co/Cu})_n \rightarrow 120\%$ MR at low T and $(\text{Fe/Cr})_n \rightarrow 220\%$ at low T.

Two Resistor Model of GMR

The GMR effect can be qualitatively explained using a two-resistor model that is based on two current models of Mott.

- key concept is to realize that there is spin-dependent conductance/resistance in a magnetic material.

why? Because the band structure is different for spin-up and spin-down electrons. For eg. a typical 3d band structure is as follows:-



- d bands are more localized i.e. the spread in energy is low and are spin-split due to exchange interactions. (Review the Stoner model)

- s band is more free-electron like

$$P = \frac{m^2}{n e^2 Z} \propto \frac{2\pi}{h} \langle |f| v |i \rangle^2 D(E_F) \rightarrow \text{Fermi's Golden rule}$$

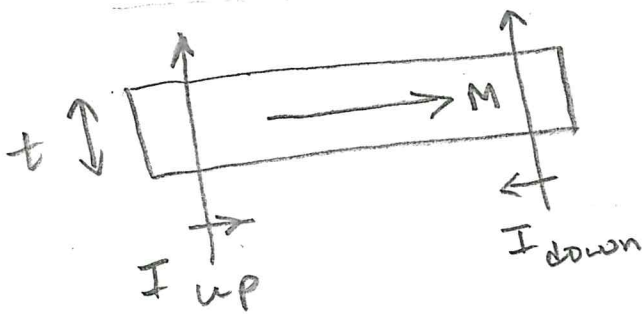
Resistivity for an up electron is different than a down electron since the scattering probabilities are different for majority and minority due to different density of states

$$\rho_{\uparrow} \propto D_{\uparrow}(E_F) \text{ and } \rho_{\downarrow} \propto D_{\downarrow}(E_F) \text{ among other factors.}$$

Therefore $\sigma_{\text{total}} = \sigma_{\uparrow} + \sigma_{\downarrow} \rightarrow$ Two-current model

$$\text{or } \frac{1}{\rho_{\text{total}}} = \frac{1}{\rho_{\uparrow}} + \frac{1}{\rho_{\downarrow}} \Rightarrow \frac{1}{R_{\text{total}}} = \frac{1}{R_{\uparrow}} + \frac{1}{R_{\downarrow}}$$

Let's calculate the resistance of an up & dn electron as it travels within a magnetic material thin film

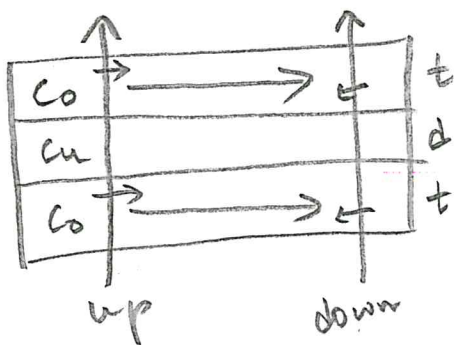


up-electrons, by definition, are electrons whose moments are parallel to the magnetization M
 down-electrons have moments anti-parallel to magnetization.

$$R = \frac{\rho t}{A} \Rightarrow R_{\uparrow} = \frac{\rho_{\uparrow} t}{A} \text{ and } R_{\downarrow} = \frac{\rho_{\downarrow} t}{A}$$

$t \rightarrow$ thickness
 $A \rightarrow$ Area.

For a GMR multilayer in parallel configuration (neglecting the resistance of the spacer layer)



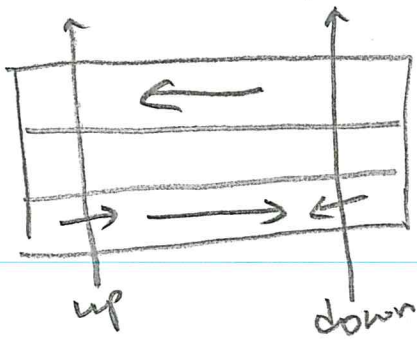
$$R_{\uparrow} = \frac{2 \rho_{\uparrow} t}{A} \text{ and } R_{\downarrow} = \frac{2 \rho_{\downarrow} t}{A}$$

$$\Rightarrow \frac{1}{R_P} = \frac{A}{2t} \left(\frac{1}{\rho_{\uparrow}} + \frac{1}{\rho_{\downarrow}} \right) \Rightarrow R_P = \frac{2t}{A} \left(\frac{\rho_{\uparrow} \rho_{\downarrow}}{\rho_{\uparrow} + \rho_{\downarrow}} \right)$$

\rightarrow Parallel

We neglected the resistance of spacer layer since $\rho_{\text{Cu}} < \rho_{\text{Co}}$, but it can be included. Typically GMR effect will reduce because of the resistance of space layer.

For an anti parallel configuration



$$R_{\uparrow} = R_{\downarrow} = \frac{(P_{\uparrow} + P_{\downarrow})t}{A}$$

$$\Rightarrow \frac{1}{R_{AP}} = \frac{2A}{(P_{\uparrow} + P_{\downarrow})t} \Rightarrow R_{AP} = \frac{(P_{\uparrow} + P_{\downarrow})t}{2A}$$

$$R_{AP} - R_P = \frac{t}{A} \left[\frac{(P_{\uparrow} + P_{\downarrow})}{2} - \frac{2P_{\uparrow}P_{\downarrow}}{P_{\uparrow} + P_{\downarrow}} \right] = \left[\frac{(P_{\downarrow} + P_{\downarrow})^2 - 4P_{\uparrow}P_{\downarrow}}{2(P_{\uparrow} + P_{\downarrow})} \right] \frac{t}{A}$$

$$= \frac{t(P_{\uparrow} - P_{\downarrow})^2}{2A(P_{\uparrow} + P_{\downarrow})}$$

Define GMR ratio = $\frac{R_{AP} - R_P}{R_P} = \frac{\frac{t}{A} \frac{(P_{\uparrow} - P_{\downarrow})^2}{2(P_{\uparrow} + P_{\downarrow})} \times \frac{A}{2t} \frac{(P_{\uparrow} + P_{\downarrow})}{P_{\uparrow}P_{\downarrow}}}{\frac{(P_{\uparrow} + P_{\downarrow})t}{2A}}$

$$= \frac{(P_{\uparrow} - P_{\downarrow})^2}{4P_{\uparrow}P_{\downarrow}}$$

Define $\alpha = \frac{P_{\downarrow}}{P_{\uparrow}}$, $GMR = \frac{P_{\uparrow}^2(1-\alpha)^2}{4P_{\uparrow}P_{\downarrow}} = \frac{(\alpha-1)^2}{4\alpha}$

$$\boxed{\text{GMR ratio} = \frac{(\alpha-1)^2}{4\alpha}}$$

$\alpha \approx 7$ for Co $\Rightarrow GMR \approx \frac{36}{28} \approx 1.3 \approx 130\%$

This is indeed the maximum value observed for Co/Cu with thickness of the non-magnetic spacer layer,

$$\boxed{\text{GMR ratio} = \frac{(\alpha-1)^2}{4(\alpha + Pd/t)(1 + Pd/t)}}$$

$d \rightarrow$ thickness of spacer layer

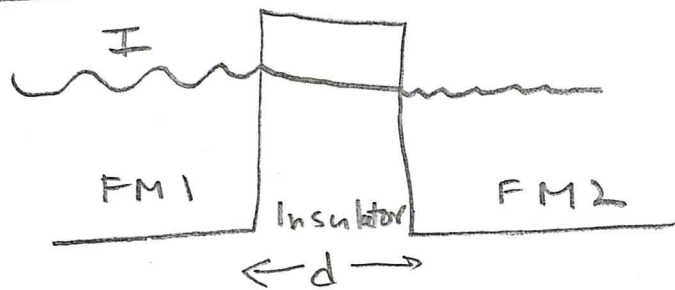
$P = \frac{P_{NM}}{P_{\uparrow}}$ NM \rightarrow Non-magnetic

Tunnel magnetoresistance

- The tunnel magnetoresistance effect is similar to GMR
- Non magnetic spacer layer in GMR is replaced by an insulator.
- The physics change dramatically, however,
- Electrons can no longer scatter through an insulator. They tunnel through.
- Therefore, resistance is caused due to the tunnel effect and not scattering \rightarrow This is the most fundamental difference between GMR & TMR.

Julliere Model of TMR

Current in an insulator is due to quantum mechanical tunneling effect.



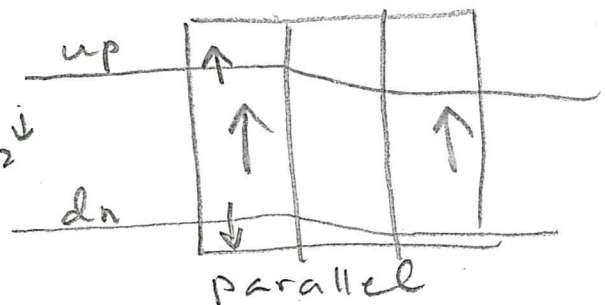
- Electrons from FM1 tunnel into empty states available in FM2.
- Tunneling current $\propto D_1 D_2$ $D_{1(2)}$ \rightarrow Density of states in 1(2)
- This relation also follows from Fermi Golden rule

$$I_{\text{total}} = I_{\text{up}} + I_{\text{down}} \quad \text{or} \quad \sigma = \sigma_{\uparrow} + \sigma_{\downarrow}$$

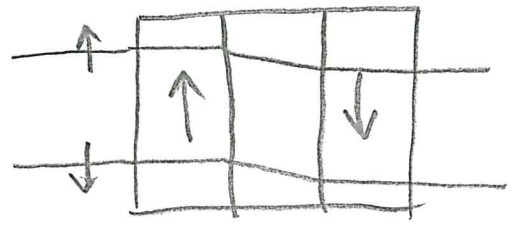
In the parallel configuration,

$$I_{\text{up}} \propto D_1^{\uparrow} D_2^{\uparrow} \Rightarrow I_p \propto D_1^{\uparrow} D_2^{\uparrow} + D_1^{\downarrow} D_2^{\downarrow}$$

$$I_{\text{dn}} \propto D_1^{\downarrow} D_2^{\downarrow}$$



Similarly, $I_{AP} \propto \underbrace{D_1^\uparrow D_2^\downarrow}_{I_{up}} + \underbrace{D_1^\downarrow D_2^\uparrow}_{I_{dn}}$



$$\frac{I_P - I_{AP}}{I_{AP}} = \frac{\sigma_P - \sigma_{AP}}{\sigma_{AP}} = \text{TMR ratio}$$

Note, $\frac{\sigma_P - \sigma_{AP}}{\sigma_{AP}} = \frac{1/\rho_P - 1/\rho_{AP}}{1/\rho_{AP}} = \frac{\rho_{AP} - \rho_P}{\rho_P} \rightarrow$ Same as GMR ratio definition.

$$= \frac{\Delta R}{R_P}$$

$$\frac{\sigma_P - \sigma_{AP}}{\sigma_{AP}} = \frac{D_1^\uparrow (D_2^\uparrow - D_2^\downarrow) + D_1^\downarrow (D_2^\downarrow - D_1^\downarrow)}{D_1^\uparrow D_2^\downarrow + D_1^\downarrow D_2^\uparrow} = \frac{(D_1^\uparrow - D_1^\downarrow)(D_2^\uparrow - D_2^\downarrow)}{D_1^\uparrow D_2^\downarrow + D_1^\downarrow D_2^\uparrow}$$

Define spin-polarization, $P = \frac{D^\uparrow - D^\downarrow}{D^\uparrow + D^\downarrow} = \frac{D^\uparrow - D^\downarrow}{D_{tot}}$ $D_{tot} = D^\uparrow + D^\downarrow$

Consider $1 - P_1 P_2 = 1 - \left(\frac{D_1^\uparrow - D_1^\downarrow}{D_1^\uparrow + D_1^\downarrow} \right) \left(\frac{D_2^\uparrow - D_2^\downarrow}{D_2^\uparrow + D_2^\downarrow} \right)$

$$= \frac{(D_1^\uparrow + D_1^\downarrow)(D_2^\uparrow + D_2^\downarrow) - (D_1^\uparrow - D_1^\downarrow)(D_2^\uparrow - D_2^\downarrow)}{(D_1^\uparrow + D_1^\downarrow)(D_2^\uparrow + D_2^\downarrow)}$$

$$= \frac{D_1^\uparrow D_2^\uparrow + D_1^\uparrow D_2^\downarrow + D_1^\downarrow D_2^\uparrow + D_1^\downarrow D_2^\downarrow - D_1^\uparrow D_2^\uparrow + D_1^\uparrow D_2^\downarrow + D_1^\downarrow D_2^\uparrow - D_1^\downarrow D_2^\downarrow}{D_{1tot} D_{2tot}}$$

$$= \frac{2(D_1^\uparrow D_2^\downarrow + D_1^\downarrow D_2^\uparrow)}{D_{1tot} D_{2tot}}$$

$$\Rightarrow D_1^\uparrow D_2^\downarrow + D_1^\downarrow D_2^\uparrow = \frac{(1 - P_1 P_2)}{2} \times D_{1tot} D_{2tot}$$

Substituting in $\frac{\sigma_P - \sigma_{AP}}{\sigma_{AP}} = \frac{(D_1^\uparrow - D_1^\downarrow)(D_2^\uparrow - D_2^\downarrow)}{D_1^\uparrow D_2^\downarrow + D_1^\downarrow D_2^\uparrow} = \frac{2(D_1^\uparrow - D_1^\downarrow)}{D_{1tot}} \frac{(D_2^\uparrow - D_2^\downarrow)}{D_{2tot}} \frac{1}{(1 - P_1 P_2)}$

$$\Rightarrow \boxed{\text{TMR} = \frac{2 P_1 P_2}{1 - P_1 P_2}}$$

Jullier's model.

when $P_1 = P_2 = 1$ $\text{TMR} \rightarrow \infty$ ideal switch